Enhancing Matching in Math Directed Reading Programs

Hamed Hekmat, Gavin McDonell

Abstract

Math directed reading programs (DRPs) are mentorship organizations that allow undergraduate students to gain exposure to concepts outside of their official math coursework under the supervision of graduate students. These programs are intended to broaden participation in math, give undergraduates a taste of advanced topics, and provide graduate students with mentorship experience. As undergraduate-graduate mentor matching is one-to-one, ensuring that the pairings are good fits is imperative to a DRP's success. Typically, matching is done by an organizing committee of graduate students without a formal system or mechanism. As an alternative to the current matching process, we consider a mechanism that prompts undergraduates for their preferences and then finds a maximum weight bipartite matching. Analyzing synthetic data from Stanford University's DRP, we find that this mechanism leads to welfare gains under two metrics. We investigate properties of this mechanism.

1. Introduction

Math directed reading programs (DRPs) — organizations in which undergraduates are paired with graduate students to gain exposure to advanced math topics — are a relatively recent development at universities in the United States. The first program of this kind was founded by Mark Behrens and Moon Duchin at the University of Chicago in 2003.[1] Since then, over 20 DRPs have sprung up across the country, including at top institutions such as Harvard University, Massachussetts Institute of Technology, and Stanford University. There is now a national DRP network that provides resources on starting programs and best practices for running them. Although there is some variation in how DRPs are implemented, the national network advises that every program feature the following:

- a one-on-one pairing between undergraduate and graduate students
- weekly meetings with mentor-mentee pairs

[1] https://sites.google.com/view/drp-network/home

- a semester (or quarter) long reading, not research, project on a topic of mutual interest not typically seen in a class
- no written testing or formal evaluation
- a final presentation done by the undergraduate to peers and mentors or the math department community
- facilitation by a graduate student organizing committee with modest faculty oversight.[2]

With these shared characteristics, DRPs have grown in popularity over the last two decades. For undergraduates interested in learning advanced math, the DRP provides a low-stakes opportunity to study high-level concepts and meet more experienced mathematicians. The individualized aspect of the program is often of great appeal. For graduate students, many of whom will seek academic positions after earning their Ph.D., the DRP allows them to gain valuable mentoring and teaching experience. Some DRPs also incentivize graduate student participation by providing compensation in the form of an honorarium. Other DRPs supply graduate students with food, textbooks, or summer funding. Ayduk (2019) found that undergraduates saw significant increases in their confidence in being able to read and understand graduate-level texts and research articles — skills that are characteristic of Ph.D. students — as a result of DRP participation. Simultaneously, graduate students gained confidence in their problem-solving abilities.

Given the rise in popularity of DRPs and the potential benefits of DRP participation, it is paramount that mentor-mentee matches be chosen carefully. We seek to improve on the current matching system.

2. Current matching process

At most universities that offer DRPs, there are more interested undergraduates than available graduate student mentors. As such, there is often an application process for undergraduates. According to the national DRP network, applications commonly prompt undergraduates for their major, courses and grades, motivation for participating in the DRP, demographic data, and math interests. Sometimes applications pose open-ended questions such as "what is something from a math class that made you curious?" or "have you ever had to explain math to someone else?" The DRP network encourages the graduate student organizing committee that designs the application to think critically about what function each application question serves.

Generally speaking, graduate student mentors need not apply to participate in the DRP. However, they may be asked to indicate their preferences for mentees. The DRP network notes that graduate student mentors might be asked what subject areas they are

[2] https://sites.google.com/view/drp-network/getting-started

interested in mentoring and if they prefer a student with an elementary, intermediate, or advanced math background. In select programs, graduate student mentors must provide a project proposal, although in most, the project is chosen by the undergraduate.

After receiving applications from undergraduates and preferences from graduate mentors, organizing committees attempt to find the best pairings. As far as we are aware, no DRP implements a formal mechanism or algorithm for its matching.

2.1 Matching at Stanford

In April 2024, the graduate student organizing committee at Stanford released the application for its spring quarter DRP. Apart from asking applicants to provide basic identification criteria (e.g., name and email), the application included seven questions:

1. What is your year in college?
2. What is your (anticipated) major?
3. Please list the math classes you have taken at Stanford or another university.
4. What is your proposed topic of study?
5. What do you hope to get out of your participation in the DRP?
6. Have you previously participated or applied to participate in the DRP?
7. If you have participated before, would you like to be paired with the same mentor. If so, who?[3]

Although the organizing committee used this application to obtain information on the undergraduates, the final pairing decisions were made somewhat randomly. From interviews with members of the committee, we learned that the final matches were chosen hastily — and unsystematically — before the deadline, with the exception of undergraduates who wanted to continue with their past mentor. Given the randomness in the current matching process in Stanford's DRP, we believe there is an opportunity to improve outcomes by implementing a new mechanism.

3. Literature review

3.1 Academic mentoring literature

Other than Ayduk's 2019 study, the academic literature on DRPs — and specifically on mentor-mentee matching in DRPs — is nonexistent. However, there is a wide body of literature on the effectiveness of academic mentoring programs and the importance of "good fit" pairings.

[3] https://mathdrp.stanford.edu/

Several studies have identified the influence that academic mentorship programs can have on students' success. In a sample of 983 first-year undergraduate students, Rodger and Tremblay (2003) found that those who received peer mentoring recorded significantly higher final grades. In a similar undergraduate peer mentoring study, Pye et al. (2016) determined that participation in a mentoring program improved students' confidence, self-efficacy, and motivation. There is also significant evidence that academic mentoring programs can improve student outcomes in science, technology, engineering, and math (STEM) disciplines. Larose et al. (2011) found that when undergraduate students in science and engineering mentored incoming STEM students, the mentees demonstrated increased motivation and adjusted better socially. Sowers et al. (2017) identified differences between mentored and non-mentored students in measures of quantitative knowledge, engagement, and career planning confidence. In math specifically, there is evidence that mentoring programs can improve students' outcomes and perceptions of the field. For example, Yue (2011) reported that incoming college freshmen who participated in a summer math mentor program showed significant improvements in problem-solving skills.

While the consensus in the literature is that academic mentoring programs can increase the likelihood of student success, there is disagreement about the optimal way to pair mentees with mentors. In particular, academics debate which mentor characteristics are most important for a pairing's success. Campbell and Campbell (2007) compared a sample of 339 undergraduates in a student-faculty mentor program against various controls and found that there was no apparent advantage associated with matching students based on gender. However, undergraduates matched with mentors of the same race had a higher average cumulative GPA and graduation rate. In contrast, Kogovšek and Ograjenšek (2019) reported that researchers with same-gender mentors recorded higher postdoctoral publication scores than their counterparts. Data from 1,013 undergraduates, graduates, and postdoctoral researchers in a study by Blake-Beard et al. (2011) indicated that both gender and race were important characteristics in academic mentoring pairings. Women and students of color in particular preferred to be matched with a mentor who shared these characteristics with them and reported receiving more help if they were matched with such a mentor. The study did not find any significant differences in academic outcomes for students who received a same-gender, same-race mentor, however. Different studies have found that factors other than shared gender and race are most important in academic mentor pairings. Hernandez et al. (2017), for instance, contended that high mentorship quality depends more on shared values than on demographic match.

Overall, the existing literature on the subject indicates that academic mentoring programs can be very valuable, but their success depends on the quality of the matches. We posit that a mechanism that allows undergraduates to express preferences for graduate mentors and determine a matching based on these preferences will lead to better pairings and a more successful DRP.

3.2 Matching literature

Our research not only builds on the existing academic mentoring literature, but also on the current literature on one-to-one matching.

In the 20th century, several fundamental contributions were made to the study of one-to-one matching. Perhaps most notably, Gale and Shapley (1962) proposed a model of two-sided, one-to-one matching in which individuals from each side hold preferences over the individuals on the other side. They created an algorithm — now known as the Gale-Shapley or deferred acceptance algorithm — for finding a stable matching and proved that when preferences are strict, there always exists a stable matching that is optimal for individuals on one side of the market. The notion of stability is important for mechanisms: if the outcome of a mechanism is stable, then there is no incentive for re-contracting nor are there grounds for legal action due to violated priorities. Interestingly, Roth (1986) proved that the sets of of unmatched individuals are the same in all stable matchings. This is known as the Rural Hospital Theorem. Further, Dubins and Freedman (1981) showed that the deferred acceptance mechanism is strategy-proof for the proposing side. However, the mechanism is not strategy-proof for the accepting side. Later, Immorlica and Mahdian (2003) proved that if each individual on the proposing side has a preference list chosen independently at random from some distribution, and each individual on the accepting side has an arbitrary complete preference list, the expected fraction of individuals on the accepting side with another stable partner vanishes as the market grows very large. Formally, if there are n individuals on each side, and $c_k(n)$ denotes the expected number of individuals on the accepting side who have more than one stable partner, then for all fixed k,

$$\lim_{n\to\infty} \frac{c_k(n)}{n} = 0.$$

This result implies that even though the deferred acceptance mechanism is not strategy-proof for the accepting side, misreporting one's preferences almost certainly cannot be beneficial if the market is very large.

Other mechanisms such as the Top Trading Cycles Mechanism (TTC) and the Efficiency Adjusted Deferred Acceptance Mechanism (EADAM) were important developments in the study of one-to-one matching. Shapley and Scarf (1974) formalized TTC, a mechanism originally developed by Gale, and showed that its outcome is in the core (i.e., in the set of feasible unblocked assignments). Roth (1982) showed that TTC is strategy-proof, and Ma (1994) proved that if preferences are strict, TTC is the only mechanism that is individually rational, Pareto efficient, and strategy-proof. Kesten (2010) developed EADAM, based on Gale and Shapley's work, to reduce efficiency costs. However, EADAM is not strategy-proof.

The deferred acceptance mechanism, TTC, and EADAM all require participants to have ordinal preferences. However, in some matching situations, this is not a reasonable assumption. For example, one group of individuals might not have enough information to rank members in the other group. Mechanisms have been proposed to address these scenarios, including those that model the matching situation as an assignment problem. The assignment problem, specifically an iteration of it known as the maximum weight bipartite matching problem, can be solved to determine the matching that maximizes some objective function. Formally, a maximum weight matching is defined as follows: given a bipartite graph $H = (V, E)$ with bipartition (X, Y) and weight function $w : E \to \mathbb{R}$, M is a maximum weight matching if

$$\sum_{e \in M} w(e) \geq \sum_{e \in M'} w(e)$$

for any possible matching M'. Kuhn (1955) developed the Hungarian algorithm to find a maximum weight bipartite matching in $O(n^4)$ time. Edmonds and Karp (1972) improved Kuhn's original algorithm to run in $O(n^3)$ time. These algorithms have wide-ranging applications and have been used in practical market design contexts. Aksoy et al. (2013), for instance, adapted the Hungarian algorithm to use in a school choice mechanism.

One last topic of interest is whether truthful play will occur for a mechanism implemented in practice. While strategy-proofness is an important quality for mechanisms, it is less valuable if participants in a mechanism do not choose their dominant strategy in practice. Vickrey (1961) formalized the second-price auction and noted that it is strategically equivalent to the ascending auction. Both auctions are strategy-proof. In practice, however, participants bid truthfully more frequently in the ascending auction than in the second-price auction. Kagel et al. (1987) ran an experiment in which subjects knew their true value and found that the predicted equilibrium occurred 76% of the time in the ascending auction but only 20% in the second-price auction. Li (2017) developed the concept of obvious strategy-proofness, which in part explains this difference. A mechanism is obviously strategy-proof if the truthful strategy is obviously dominant. That is, if S is the truthful strategy, then the worst-case outcome under S is at least as good as the best-case outcome under any other strategy S'. The ascending auction, unlike the second-price auction, is obviously strategy-proof, which makes it easier for participants to identify their dominant strategy. As theoretical strategy-proofness does not guarantee truthful play, we might ask if individuals will play truthfully in a mechanism that is not strategy-proof but appears to be incentive-compatible.

4. The DRP matching mechanism

At most universities, undergraduates are not in close enough contact with Ph.D. math students to hold strict preferences over them. Similarly, graduate student mentors are unlikely to know the undergraduate population sufficiently well to express strict preferences. Thus, we seek a mechanism that does not require undergraduate students to express strict preferences over mentors but instead allows them to indicate their desire for their mentor to embody certain characteristics. For this reason, we turn to a mechanism that involves a maximum weight matching.

4.1 Maximum weight bipartite matching

We can formulate the DRP mentor-mentee matching problem as a maximum weight problem. We assume that there are n undergraduate student applicants and m graduate student mentors where $n > m$. This is a realistic assumption for most DRPs. Specifically, let $U = \{u_1, \dots, u_n\}$ be the set of undergraduate students and $G = \{g_1, \dots, g_m\}$ the set of graduate students. We add $n - m$ elements to G such that the resulting set, G', has the same number of elements as U. Then, $G' = \{g_1, \dots, g_m, g_{m+1}, \dots, g_n\}$ and we let $V = U \cup G'$ and $E = \{(u_i, g_j) : i, j \in \{1, \dots, n\}\}$. Choosing some weight function $w : E \to \mathbb{R}$, we can write this problem as the following linear program (LP):

$$\text{maximize} \sum_{i=1}^{n} \sum_{j=1}^{n} w(u_i, g_j) x(u_i, g_j) \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} x(u_i, g_j) = 1,\ \forall u_i \in U$$

$$\sum_{i=1}^{n} x(u_i, g_j) = 1,\ \forall g_j \in G$$

$$0 \leq x(u_i, g_j) \leq 1,\ \forall (u_i, g_j) \in E$$

$$x(u_i, g_j) \in \mathbb{Z},\ \forall (u_i, g_j) \in E.$$

Note that we have created n^2 decision variables: $x(u_i, g_j)$ for $i, j = 1, \dots, n$. Solving this LP for these n^2 unknowns will give us the mentor-mentee matching that achieves the maximum total weight.

4.2 Choosing a weight function

We would like the weight function to map each edge to a real number that roughly represents the goodness of the match. Since DRP curricula can be quite specialized, it is imperative that undergraduates receive a mentor who is capable of teaching their desired math subject. The existing literature on academic mentoring indicates that gender and race can be important characteristics in mentor-mentee pairings. Further, we posit that the language(s) a mentor speaks is relevant. Thus, we prompt undergraduates to express preferences across each of these categories: preferences for the math subject(s) they would like to study, preferences for their mentor's gender and race, and preferences for the language(s) their mentor speaks. This information would be collected in a new DRP application.

For each undergraduate student u_i, we define p_i as a 4-tuple that captures the student's preferences. We denote $p_{i,k}$ as the set in the kth component of the tuple for undergraduate u_i. The first component of the tuple contains the set of math subjects that the undergraduate indicates they want to study. The second component contains the set of gender preferences, the third race preferences, and the fourth language preferences. For example, undergraduate u_i's preference tuple may resemble the following:

$$p_i = (\{\text{real analysis}\}, \{\text{female}\}, \{\text{Asian}\}, \{\text{English, Chinese}\}).$$

For each graduate student mentor g_j, we define ϕ_j as a 4-tuple that represents their profile. The ordering of the information in ϕ_j is similar to in p_i. The first component of the tuple contains the set of math subjects that the graduate mentor is both willing and able to teach. The second component contains the graduate mentor's gender, the third their race, and the fourth the language(s) they speak proficiently. For instance, graduate mentor g_j's profile tuple may look like:

$$\phi_j = (\{\text{real analysis, combinatorics, algebraic geometry}\}, \{\text{female}\}, \{\text{Black}\}, \{\text{English, French}\}).$$

We can calculate the number of areas in which an undergraduate student's preferences match a graduate mentor's profile with the sum

$$\sum_{k=1}^{4} \mathbb{I}\{|p_{i,k} \cap \phi_{j,k}| \geq 1\}$$

where $\mathbb{I}$ is an indicator variable. Using the example p_i and ϕ_j above, we compute $\sum_{k=1}^{4} \mathbb{I}\{|p_{i,k} \cap \phi_{j,k}| \geq 1\} = 3$. Further, we can incorporate how much an undergraduate values a match in each area. In the DRP application, undergraduates would be prompted to report a value representing how important a match is to them in a given area. In particular, undergraduate u_i reports a vector $c_i \in [0, 100]^4$ such that $\sum_{k=1}^{4} c_{i,k} = 100$. Now, we define a weight function $w : E \to \mathbb{R}$ as follows:

$$w(u_i, g_j) = \begin{cases} \sum_{k=1}^{4} c_{i,k} \mathbb{I}\{|p_{i,k} \cap \phi_{j,k}| \geq 1\} \text{ if } j \in \{1, \dots, m\} \\ 0 \text{ otherwise.} \end{cases}$$

Note that for any edge (u_i, g_j) where $j > m$, the weight is 0. This is because a (u_i, g_j) pairing for $j > m$ means u_i is not matched in the DRP.

Although graduate students likely hold preferences over undergraduates — or at least prefer certain characteristics in their undergraduate mentees — we do not incorporate these preferences into our weight function. The DRP aims to pair undergraduates with a mentor who suits them; it does not guarantee graduate mentors an undergraduate who suits their interests. The DRP is also meant to provide graduate students with teaching and mentoring experience. In theory, being paired with any undergraduate provides an opportunity to gain mentoring experience. Additionally, graduate students are often compensated. As such, our weight function reflects only undergraduate preferences.

In summary, we propose a mechanism that prompts undergraduates for their preferences in a graduate student mentor across four categories, computes weights on edges in a bipartite graph, and then determines a maximum weight matching with respect to this weight function.

5. Data and Simulation

We designed a simulation to compare the maximum weight matching mechanism to random assignment in Stanford's DRP. Based on our interviews with the Stanford organizing committee as discussed earlier, we believe that the current matching process resembles random assignment.

We were unable to get comprehensive application data from the Stanford Math Department. Thus, we developed synthetic datasets, sampling applicant preference submissions and mentor profiles from distributions for the relevant categories. We leveraged historical DRP data to estimate how many students and mentors we needed to model and created a distribution for general subject area interest from previous quarters' matches.[4] As for race and gender distributions, we utilized figures reported on the official Stanford undergraduate and graduate student profile pages.[5] Our language distributions were based on U.S. Census Bureau data on languages spoken at home in addition to English.[6]

Alongside our observations from the literature review regarding factors that contribute to a successful match, we interviewed current DRP participants to get a sense of how student preference weights might be distributed across the four categories. Based on these interviews, we chose to sample each undergraduate's preference weights from uniform distributions for the four areas and normalize them such that the sum of the weights was equal to 100. The mean preference weights were 50 for subject area, 20 for race and gender, and 10 for language. It is important to note that these distributions are based off assumptions made either due to lack of data or to simplify the model for the simulation. For example, the Stanford student profile page only contained data regarding the gender binary, so we had to model all synthetic applicants and mentors as such. Another example is the assumption that larger scale demographics match the Stanford Math Department or, more specifically, the pool of people hoping to participate in the DRP. In general, though it would be impossible to recreate perfectly representative distributions, we believe our distributions mimic the pool of applicants and mentors well enough to demonstrate the effectiveness of the maximum weight matching mechanism.

After sampling from the aforementioned distributions to create 25 applicant preference tuples, corresponding preference weight vectors, and 17 mentor profiles, we used Python's

[4] https://mathdrp.stanford.edu/past-participants.html

[5] https://facts.stanford.edu/academics/undergraduate-profile/
https://facts.stanford.edu/academics/graduate-profile/

[6] https://data.census.gov/table/ACSDT1Y2022.B16001?text=Language&t=Language%20Spoken%20at%20Home

networkx library to generate a bipartite graph.[7] We computed edge weights for every possible applicant-mentor pairing and determined the number of categories where there was overlap between an applicant's preferences and a mentor's profile using the same expression described above:

$$\sum_{k=1}^{4} \mathbb{I}\{|p_{i,k} \cap \phi_{j,k}| \geq 1\}.$$

This provides us with an additional metric to assess the quality of a specific matching.

To find the maximum weight matching given the synthetic dataset, we applied networkx's built-in maximum weight matching function that utilizes the $O(n^3)$ algorithm developed by Edmunds and Karp. As previously noted, we chose to model the current DRP matching process as random assignment. For a synthetic dataset generated in the manner described above, we found both the maximum weight matching and 10,000 random matchings to obtain an average. We calculated the total weight of each matching as well as the total number of categories with overlap between applicant preferences and mentor profiles across all pairings in the matching. For robustness, we repeated this process for 100 randomly generated synthetic datasets, taking the average value of our two metrics across the maximum weight matchings and the random matchings.

6. Analysis

6.1 Simulation Results

Across 100 synthetic datasets and 10,000 iterations per dataset, the simulation showed that the average total weight for a random matching was 974.92, and the average total number of categories with overlap was 32.01. This translates to per-applicant values of 39.00 and 1.28, respectively, assuming that unmatched applicants have an edge weight of 0 as well as 0 categories with overlap in the final matching. Across the same 100 synthetic datasets, the resulting average total weight for a maximum weight matching was 1,561.65, and the average number of categories with overlap was 49.88. This translates to per-applicant values of 62.47 and 2.00, respectively.

We observe that our proposed maximum weight matching mechanism increases social welfare under both metrics. The average edge weight in the maximum weight matching is more than 1.5 times the average edge weight in the random matching, which is significant considering the weight reflects student preferences. It is also critical that the maximum weight matching leads to an increased number of total categories with overlap. Per our literature review, we note that overlap in our chosen categories may play a significant role in determining match quality and thus student satisfaction. This is key in encouraging future student engagement in math, the intended goal of DRPs.

The social welfare gains are even more promising when paired with the added ad-

[7] https://networkx.org/. 25 applicants and 17 mentors is based on historical data from DRP matches.

vantage of efficiency gains. The mechanism was straightforward to implement and runs extremely quickly, meaning its utilization has the potential to save the organizing committee a significant amount of time. After the initial overhead of making some design choices regarding application questions and preference and profile categories, the algorithm will nearly instantaneously output a matching that has the potential to increase social welfare across the applicant pool.

6.2 Pareto efficiency

We claim that our maximum weight matching mechanism is Pareto efficient.

Theorem 1. *If $w(u_i, g_j) > w(u_i, g_k)$ implies that u_i prefers to be matched with g_j more than g_k, then the outcome M of the maximum weight matching mechanism is Pareto efficient.*[8]

Proof. Let M be the outcome of the maximum weight matching mechanism. Assume for the sake of contradiction that there exists another matching M' in which no undergraduate receives a match corresponding to a lower weight, and one or more undergraduates have matches corresponding to higher weights in M' than in M. Then, M' would have a higher total weight than M, which contradicts the fact that M is the maximum weight matching. Thus, there are no Pareto improvements from M. $\square$

6.3 Strategy-proofness

We argue that our proposed mechanism is not strategy-proof.

Theorem 2. *The maximum weight matching mechanism is not strategy-proof.*

Proof. To illustrate that the mechanism is not strategy-proof, we consider the following counterexample with two students and two mentors. Suppose $U = \{u_1, u_2\}$ and $G = \{g_1, g_2\}$, and that the undergraduate true preferences and graduate student profiles are as follows:

$$p_1 = (\{\text{real analysis}\}, \{\text{female}\}, \{\text{Asian}\}, \{\text{English, Chinese}\}), c_1 = (25, 25, 25, 25)$$

$$p_2 = (\{\text{graph theory}\}, \{\text{female}\}, \{\text{Black}\}, \{\text{English}\}), c_2 = (20, 25, 30, 25)$$

$$\phi_1 = (\{\text{real analysis, combinatorics, algebraic geometry}\}, \{\text{female}\}, \{\text{Black}\}, \{\text{English, French}\})$$

$$\phi_2 = (\{\text{Fourier analysis}\}, \{\text{male}\}, \{\text{White}\}, \{\text{English, Spanish}\}).$$

Then, using that $w(u_i, g_j) = \sum_{k=1}^{4} c_{i,k} \mathbb{I}\{|p_{i,k} \cap \phi_{j,k}| \geq 1\}$, we calculate that $w(u_1, g_1) = 75, w(u_1, g_2) = 25, w(u_2, g_1) = 80$, and $w(u_2, g_2) = 25$. Based on these weights, we can see that the maximum weight matching is $M = \{(u_1, g_2), (u_2, g_1)\}$. We assume that

[8] It is likely that if $w(u_i, g_j) > w(u_i, g_k)$, u_i prefers to be matched with g_j more than g_k, but this assumption is not trivial.

because $w(u_1, g_1) < w(u_1, g_2)$, undergraduate 1 prefers to be matched with graduate mentor 2. Now, suppose that undergraduate 1 misreports that $c_1 = (100, 0, 0, 0)$ and that undergraduate 2 still reports truthfully. Now, $w(u_1, g_1) = 100, w(u_1, g_2) = 0$, and the maximum weight matching is $M' = \{(u_1, g_1), (u_2, g_2)\}$. This is a better pairing for undergraduate 1, which illustrates that the maximum weight matching mechanism is not strategy-proof. □

However, we believe that it is unlikely that undergraduates would misreport their preferences. We expect that undergraduates who participate in the mechanism will think that reporting truthfully increases the likelihood that they will be matched with a suitable mentor. Further, we believe that it will be difficult for undergraduates to identify and choose an untruthful strategy that will likely benefit them. This is especially true considering the average participation in the Stanford DRP is 1.42 quarters, meaning undergraduates have very few opportunities to experiment with various strategies and learn about the mechanism.[9] Without experimental evidence, we cannot verify these claims. They could be tested in an experiment that mirrors Kagel et al. (1987), in which subjects know their true preferences (i.e., p_i and c_i).

6.4 Strategy-proofness in the large

As noted in the literature review, there are mechanisms that become effectively strategy-proof as the number of participants becomes very large. We investigate whether this is true for the maximum weight matching mechanism. Specifically, we consider the situation in which there are n undergraduates and n graduates, and each $w(u_i, g_j)$ is chosen independently from $U(0, 1)$ for $i, j = 1, \ldots, n$. Let $c(n)$ denote the expected number of undergraduates who do not receive their optimal match. We seek to compute

$$\lim_{n \to \infty} \frac{c(n)}{n}. \tag{2}$$

If the limit (2) is equal to zero, then it implies that dishonesty almost certainly cannot benefit a player.

To illustrate, we first compute $c(2)/2$. Suppose $U = (u_1, u_2)$ and $G = (g_1, g_2)$, which means there are two possible matchings: $M_1 = \{(u_1, g_1), (u_2, g_2)\}$ and $M_2 = \{(u_1, g_2), (u_2, g_1)\}$. For notational simplicity, we let e_{ij} denote the weight on edge (u_i, g_j).

[9] https://mathdrp.stanford.edu/past-participants.html

Each e_{ij} is drawn independently from $U(0,1)$.

$$\begin{aligned}
c(2) &= \sum_{i=1}^{2}\sum_{j=1}^{2} \mathbb{P}(u_i \text{ does not receive optimal match} | M_j) \\
&= 4 \cdot \mathbb{P}(u_1 \text{ does not receive optimal match} | M_1) \\
&= 4 \cdot \frac{\mathbb{P}(u_1 \text{ does not receive optimal match} \cap M_1)}{\mathbb{P}(M_1)} \\
&= 8 \cdot \mathbb{P}(e_{11} < e_{22}) \cdot \mathbb{P}(M_1 | e_{11} < e_{12}) \\
&= 4 \cdot \mathbb{P}(e_{11} + e_{22} > e_{12} + e_{21} | e_{11} < e_{12}) \\
&= 4 \cdot \mathbb{P}(e_{22} - e_{21} > e_{12} - e_{11} | e_{12} - e_{11} > 0) \\
&= 4 \cdot \int_0^1 \int_0^x (1-x)(1-y) dy dx \\
&= \frac{1}{2}
\end{aligned}$$

Thus, $c(2)/2 = 1/4$. Note that in the calculation above, $\mathbb{P}(M_i)$ refers to the probability that M_i is the maximum weight matching. While the $n = 2$ case can be calculated with relative ease, the general case is significantly more complicated. Following the procedure above, we determine that

$$\frac{c(n)}{n} = \frac{(n!)^2}{2^{n-1}} \cdot \phi(n), \text{ where } \phi(n) = \mathbb{P}\left(M_1 | \bigcup_{j=1}^{n} e_{11} < e_{1j}\right). \tag{3}$$

We approximate $c(n)/n$ using a simulation. For each n, we compute the number of undergraduates who do not receive their optimal match in each of 10,000 trials. The average number who do not receive their best match divided by n is roughly $c(n)/n$. These approximate values are reported in the table below:

n	2	3	4	5	...	10	...	50	...	100	...	200
$\frac{c(n)}{n}$	0.255	0.321	0.363	0.392		0.443		0.487		0.494		0.497

From these observations, it appears that as n increases, so does $c(n)/n$, with possible convergence to 1/2. If $lim_{n\to\infty} c(n)/n = 1/2$, this would imply that as the matching pool grows very large, half of undergraduates would receive their optimal match. As deviating to a non-truthful strategy would not always guarantee a better outcome, we believe that undergraduates would be incentivized to be truthful if n is large. Even if n is between 15 and 30, a reasonable assumption for the DRP, $c(n)/n$ is near 1/2.

7. Conclusion

This paper has identified the need for improvement in the DRP matching system and proposed a maximum weight matching mechanism to address these issues. Illustrated by simulation results using synthetic data, the maximum weight matching mechanism performs better than the current system under two key metrics: total weight (a function of undergraduates' preferences) and categories of overlap. Moreover, the maximum weight matching has the potential to save the DRP organizing committee time and resources. Under our assumptions, the maximum weight matching is Pareto efficient and not strategy-proof, although we suspect that few undergraduates would deviate from truthfully reporting their preferences. Further research could entail simulating the maximum weight matching mechanism with real data and investigating the behavior of the maximum weight matching mechanism in the limit.

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